

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



General Certificate of Education
Advanced Level Examination
June 2011

Mathematics

MFP4

Unit Further Pure 4

Wednesday 22 June 2011 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.



J U N 1 1 M F P 4 0 1

Answer **all** questions in the spaces provided.

1 The matrices **A** and **B** are given in terms of p by

$$\mathbf{A} = \begin{bmatrix} 1 & p & 4 \\ -3 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} p & 1 & 5 \\ 9 & p & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

(a) Find each of $\det \mathbf{A}$ and $\det \mathbf{B}$ in terms of p . (3 marks)

(b) Without finding \mathbf{AB} , determine all values of p for which \mathbf{AB} is singular. (3 marks)

QUESTION
PART
REFERENCE

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....



QUESTION
PART
REFERENCE

A large rectangular area with a vertical line on the left side and horizontal dotted lines across the rest, providing a space for writing answers.

Turn over ►



2

The plane transformation T is the composition of a reflection in the line $y = x \tan \alpha$ followed by an anticlockwise rotation about O through an angle β .

Determine the matrix which represents T , and hence describe T as a single transformation.

(6 marks)

QUESTION
PART
REFERENCE

Dotted lines for writing the answer.



QUESTION
PART
REFERENCE

Area containing horizontal dotted lines for writing answers.

Turn over ►



3 Given the vectors $\mathbf{p} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} 2 \\ 3 \\ t \end{bmatrix}$, where t is a scalar parameter, determine the value of t in each of the following cases:

(a) $\mathbf{p} \times \mathbf{q}$ is parallel to \mathbf{r} ; (3 marks)

(b) \mathbf{p} , \mathbf{q} and \mathbf{r} are linearly dependent. (3 marks)

QUESTION
PART
REFERENCE



4 The system of equations S is given in terms of the real parameters a and b by

$$2x + y + 3z = a + 1$$

$$5x - 2y + (a + 1)z = 3$$

$$ax + 2y + 4z = b$$

(a) Find the two values of a for which S does not have a unique solution. (4 marks)

(b) In the case when $a = 2$, determine the value of b for which S has infinitely many solutions. (4 marks)

QUESTION
PART
REFERENCE

A large area for writing the answer, consisting of a vertical line on the left and horizontal dotted lines across the rest of the space.



QUESTION
PART
REFERENCE

Handwriting practice area with a vertical margin line on the left and horizontal dotted lines for writing.

Turn over ▶



5 (a) (i) Find the eigenvalues and corresponding eigenvectors of $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ -2 & 8 \end{bmatrix}$. (6 marks)

(ii) Hence write down each of the matrices \mathbf{U} , \mathbf{D} and \mathbf{U}^{-1} such that $\mathbf{A} = \mathbf{UDU}^{-1}$, where \mathbf{D} is a diagonal matrix. (4 marks)

(b) A 2×2 matrix \mathbf{M} has distinct real eigenvalues λ and μ , with corresponding eigenvectors \mathbf{v}_1 and \mathbf{v}_2 .

(i) By considering the diagonalised form of \mathbf{M} , determine the eigenvalues of \mathbf{M}^3 . (2 marks)

(ii) Write down the eigenvectors of \mathbf{M}^3 . (1 mark)

QUESTION
PART
REFERENCE

A large rectangular area with horizontal dashed lines, intended for student answers.



6 (a) The transformation U of three-dimensional space is represented by the matrix

$$\begin{bmatrix} 1 & 4 & -3 \\ 2 & -1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

(i) Write down a vector equation for the line L with cartesian equation

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{6} \quad (2 \text{ marks})$$

(ii) Find a vector equation for the image of L under U , and deduce that it is a line through the origin. (4 marks)

(b) The plane transformation V is represented by the matrix $\begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}$.

L_1 is the line with equation $y = \frac{1}{2}x + k$, and L_2 is the image of L_1 under V .

(i) Find, in the form $y = mx + c$, the cartesian equation for L_2 . (4 marks)

(ii) Deduce that L_2 is parallel to L_1 and find, in terms of k , the distance between these two lines. (3 marks)

QUESTION
PART
REFERENCE



QUESTION
PART
REFERENCE

Handwriting practice grid consisting of horizontal dotted lines within a rectangular border.

Turn over ►



1 5

$$7 \quad \text{Let } \Delta = \begin{vmatrix} n(n+1) & n+1 & -1 \\ 0 & 1 & n \\ 1 & -(n+1) & 1 \end{vmatrix}.$$

(a) (i) Show that $(n^2 + n + 1)$ is a factor of Δ . (2 marks)

(ii) Hence, or otherwise, express Δ in factorised form. (2 marks)

(b) By expanding Δ directly, show that

$$\Delta = [n(n+1)]^2 + f(n)$$

where $f(n)$ can be expressed as the sum of two squares. (2 marks)

(c) Hence express the number 12 321 as the sum of three squares. (2 marks)

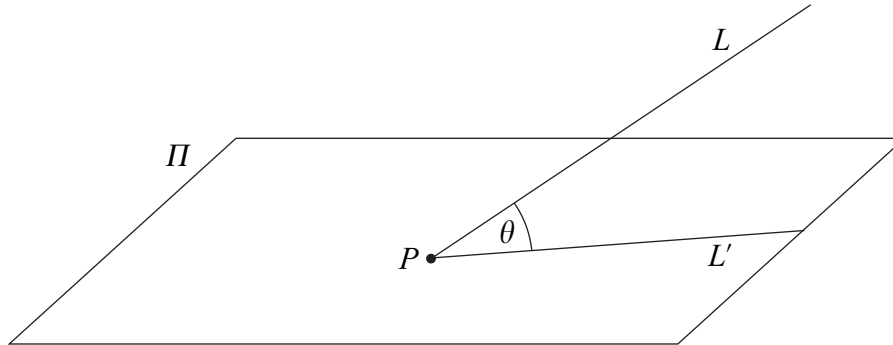
QUESTION
PART
REFERENCE



- 8** The diagram shows the plane Π and the lines L and L' . The plane Π and the line L have equations

$$\mathbf{r} \cdot \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix} = 37 \quad \text{and} \quad \mathbf{r} = \begin{bmatrix} 1 \\ 2 \\ -7 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

The line L does not lie in Π , and intersects it at the point P .



- (a) Determine the value of θ , the angle between L and Π , giving your answer to the nearest 0.1° . (4 marks)
- (b) Find the coordinates of P . (4 marks)
- (c) The line L' lies in Π and is such that the angle between L and L' is θ , the angle between L and Π .
- (i) Find a vector which is parallel to Π and perpendicular to L . (3 marks)
- (ii) Hence, or otherwise, find a vector equation for L' in the form $\mathbf{r} = \mathbf{a} + \mu\mathbf{b}$. (4 marks)

QUESTION
PART
REFERENCE

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



QUESTION
PART
REFERENCE

Turn over ►



1 9

QUESTION
PART
REFERENCE

Area with horizontal dotted lines for writing.

END OF QUESTIONS

Copyright © 2011 AQA and its licensors. All rights reserved.

